(Pages : 3) **8736**

Reg. No. : ....

Name : ....

Fourth Semester B.Tech. Degree Examination, February 2015
(2008 Scheme)
ELECTRONICS AND COMMUNICATION
08.401 : Engineering Mathematics III (TA)
Probability and Random Processes
(Special Supplementary)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** questions. **Each** question carries **4** marks. :

1. The probability density function of a random variable X is given by $f(x) = c(4x - 2x^2)$ for $0 < x < 2$ and 0 otherwise. What is the value of c. Find $P[0 < X < 2]$.
2. If x follows a Gaussian distribution N (3, 2), find the value of 'k' such that $P[\,|X - 3| > k] = 0.05$.
3. If Xi, i = 1, 2, ..., 10 be independent random variables each being uniformly distributed over (0, 1), estimate $P[\sum_{i=1} Xi > 7]$.
4. The joint probability function of (X, Y) is $f(x, y) = (8/9)xy$ for $0 < x < y < 2$ and 0 otherwise. Find the marginal density functions of X and Y.
5. A random process X(t) is defined as X(t) = A cos (wt + q) where w and q are constants and A is a random variable. Determine whether X(t) is WSS or not.
6. Find the mean and variance of the stationary process {X (t)} whose autocorrelation function is given by $R(\tau) = 16 + 9/(1 + 6\tau^2)$.
7. A hospital receives on an average 3 emergency calls in a 10 minute interval. What is the probability that there are at most 3 emergency calls in 10 minute interval.

P.T.O.

8. Check whether the following stochastic matrix is regular or not.

$$P = \begin{matrix} 0 & 1 \\ 1/2 & 1/2 \end{matrix}.$$

9. State Weiner- Khinchine theorem.

10. Define a Gaussian process. State any two properties of a Gaussian process.

PART – B

Answer **one** question from **each** Module. **Each** question carries **20** marks.

Module – I

11. a) If the pdf of a random variable X is given by $f(x) = kx^2 e^{-x}$ for $0 < x < \alpha$, find the value of k and also the mean and variance X.

b) If a fair coin is tossed at random 5 independent times, find the conditional probability of 5 heads given that there are at least 4 heads.

c) Find the mean and variance of a random variable X which is uniformly distributed in [a, b].

12. a) The average marks in an examination of a particular class is 79. The standard deviation is 5. If the marks are distributed normally, how many students in a class of 200 did not receive marks between 75 and 85.

b) The joint probability density function of X and Y is given by

$f(x, y) = kxy\, e^{-(x^2+y^2)},\ x, y > 0$. Find the value of 'k' and prove that X and Y are independent.

Module – II

13. a) Describe SSS and WSS processes with suitable examples.

b) Define the autocorrelation and autocovariance of a random process.

c) Find the autocorrelation of the random process $X(t) = R\cos(\omega t + \phi)$ where R and ϕ are independent random variables and ϕ is uniformly distributed in $(-\pi, \pi)$.

14. a) The number of accidents in a city follows a Poisson process with mean 2 per day and the number Xi of people involved in the i^{th} accident has the distribution (independent) $P(Xi = k) = 1/2^k$, $k > 0$. Find the mean and variance of the number of people involved in accidents per week.

b) Let the probability distribution of the process $\{X(t)\}$ is given by

$P(X(t) = n) = (at)^{n-1} / (1 + at)^{n+1}$ for $n = 1, 2, 3, \ldots$ and $(at)/(1 + at)$ for $n = 0$. Show that $\{X(t)\}$ is not stationary.

Module – III

15. a) Define a homogeneous Markov chain.

b) A man goes to his office by car or catches a train every day. He never goes 2 days in a row by train but if he drives one day then the next day he will go by car or train with equal probability. Now suppose that on the first day of the week the man tossed a fair die and went by car to work if and only if a '6' appeared. Find :

i) The probability that he went by train on the third day and

ii) The probability that he went by car to work in a long run.

16. a) Let $X(t) = A \cos wt + B \sin wt$ be a process where A and B are uncorrelated random variables with zero mean and equal variance. Show that X(t) is mean-ergodic.

b) Find the power spectral density of a WSS process with autocorrelation function

$R(\tau) = e^{-\alpha\tau^2}$.

c) The integral $Y(t) = \int_0^t v(u)du$ is the output of a linear system with input $X(t) = v(t).\ U(t)$ and impulse response $h(t) = U(t)$ where v(t) is white noise with average intensity q(t). Find the average power of the output.
